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Time-Displaced Correlation Functions in an Infinite One-Dimensional Mixture of Hard Rods with Different Diameters

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Time-displaced conditional distribution functions are calculated for an infinite, one-dimensional mixture of equal-mass hard rods of different diameters. The kinetic equation that describes the time dependence of the one-particle total distribution function is found to be non-Markovian, in contrast with the situation in systems of identical rods. The correlation function does not contain any isolated damped oscillation, except for systems of equal-diameter rods with discrete velocities. Thus, we generalize the one-component results of Lebowitz, Percus, and Sykes, removing some nontypical features of that system.

KEY WORDS: Time-displaced correlation functions; mixture; hard rods; different diameters; one dimension.

1. INTRODUCTION

Some aspects of the complicated dynamical behavior of real many-particle systems can, one hopes, be understood from the analysis of simple models. In the absence of exactly solvable systems of higher complexity, some effort has been put into the study of a one-dimensional system of hard rods. Following Jepsen's work[1] on the velocity autocorrelation function, Lebowitz, Percus, and Sykes (LPS)[2,3] obtained the exact time-dependent self-

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$[f_s(r, v; t, v')]$ and total $[f(r, v; t, v')]$ one-particle conditional distributions for this system. These time-displaced distribution functions (tdf) are directly related to the van Hove correlation functions[4] whose Laplace–Fourier transform is measured in neutron and X-ray scattering experiments on fluids. LPS also formulated and investigated the kinetic equations governing the time evolution of the tdf. This led them to the derivation of approximate kinetic equations for time-dependent correlation functions of three-dimensional fluid systems[8] whose solutions are in qualitative agreement with experimental results. Recently these equations have been developed further to yield semiquantitative agreement with computer results.[9]

In this paper we shall derive the exact time-dependent correlation functions for an infinite, one-dimensional mixture of hard-rod particles of equal masses with densities ρ_σ and diameters d_σ, $\sigma = 1,..., S$. The presence of varying diameters removes some of the peculiarly simple and nontypical features of the one-component hard-rod system. The dissipation of information is now more complex, possibly affecting also the ergodic behavior of the system: e.g., while the one-component infinite system can be shown (under some mild restrictions) to be Bernoulli,[5,6] the mixture is so far known only to be a K-system.[7] In this, more general, setup the function $f(r, v; t, v')$ does not obey a Markovian kinetic equation, in contrast with the (surprising) Markovian behavior of this function in a system of rods of equal diameters.[3]

2. THE DYNAMICAL SYSTEM

The effect of a collision of equal-mass hard rods is merely an exchange of the velocities. One may therefore discuss the dynamics by focusing attention on the "velocity pulses," which move at constant velocity except for moments of collision, when they jump from one rod to another. To be specific, we shall refer to the midpoint as the position of a rod and that of the corresponding velocity pulse.

The *self-distribution function* $f_s(q, v, t\,|\,\bar{q}, \bar{v}, \bar{t})$ is the probability density, in an equilibrium state, of finding at the time t a "test particle" at the position q with velocity v, given that this particle had the position and velocity $(\bar{q}, \bar{v})$ at the time $\bar{t}$. While in systems with more general interactions f_s will also depend on σ, a little thought shows (cf. also Refs. 1 and 2) that it does not in our system. The motion of a single particle through the system, in an equilibrium state, is affected by the size of the diameters only through the reduced density: $n = \rho/(1 - \rho\langle d\rangle)$ (which, together with the velocity distribution, determines the rate of collisions). Here $\langle d\rangle = \sum \rho_\sigma d_\sigma/\rho$ is the average of the diameters and $\rho = \sum \rho_\sigma$ is the density. It follows therefore that the distribution function is the same as the one given in Refs. 1 and 2 for a system of equal rods.

A related quantity, whose dependence on the distribution of the diameters is less trivial, is the self-distribution function of a velocity pulse. We shall denote it by $g(q, v, \sigma, t \mid \bar{q}, \bar{v}, \bar{\sigma}, \bar{t})$, using σ for the one-particle variable that determines the size of its diameter. For a given velocity pulse σ will refer to the particle on which the pulse is "riding" at the given time. We shall compute this function in the next section.

Another quantity that we compute is the *total distribution function* $f(q, v, \sigma, t \mid \bar{q}, \bar{v}, \bar{\sigma}, \bar{t})$—the conditional probability density, in an equilibrium state, of finding, at the time t, a particle with position, velocity, and diameter (q, v, σ) given that at the time $\bar{t}$ there is a particle with the phase $(\bar{q}, \bar{v}, \bar{\sigma})$.

For the precise formulation of the problem we use the following notation.

$\Gamma = \mathbb{R} \times \mathbb{R} \times \mathrm{S}$ is the one-particle phase space whose points represent values of (q, v, σ). The number of elements of S equals the number of different components of the mixture.

Ω, the phase space of the system in the thermodynamic (infinite-size) limit, is the space of configurations, i.e., of equivalence classes, modulo permutations, of sequences $\{(q_i, v_i, \sigma_i)\}_{i \in \mathbb{Z}}$ with $(q_i, v_i, \sigma_i) \in \Gamma$, $\forall i \in \mathbb{Z}$.

The *local symmetric observables* are represented by functions on Ω that depend (measurably) on the phases (positions, velocities, and diameters) of only those particles that occupy some specified bounded region of $\mathbb{R}$. Examples of such observables are the numbers of particles of different species in [0, 1]. (These observables generate a "quasilocal" algebra.)

States of the infinite system are given by probability measures on (Ω, X) and *expectation values* of observables are obtained by integration. Equilibrium phenomena are described by Gibbs (equilibrium) states which are the proper limits of finite-volume grand canonical ensembles.[12]

Since the collection of velocities is unchanged by the collisions, our system certainly admits additional states that are invariant under the time evolution and space translations and have good clustering properties. In these states the distribution of the particles on the line, parametrized by the densities ρ_σ, is the same as in the corresponding equilibrium states. The velocities of the particles are independently distributed with a probability distribution of density $h(v)$. For Gibbs states $h(v) = [\exp(-\beta m v^2/2)]/(2\pi/\beta m)^{1/2}$.

We shall now compute the time-displaced correlations for states of the above type. Our results will also apply, with some straightforward modifications, to systems in which the diameters vary continuously.

3. THE PULSE SELF-DISTRIBUTION FUNCTION

3.1. Relevant Phase Space

The phase space that describes the system with a singled-out "test" velocity pulse is $\Gamma \times \Omega$. We are interested in states that are initially described by some probability measure $d\mu$ on Γ (which describes the state of the test particle) and have, for any given $(\bar{q}, \bar{v}, \bar{\sigma}) \in \Gamma$, the rest of the system distributed by the equilibrium conditional probability on Ω, i.e., the rest of the system is in equilibrium in an external potential created by the test particle at $\bar{q}$. We shall call these *uncorrelated states.*

Starting from an uncorrelated state, the distribution of the test particle at time t depends linearly on its initial distribution $d\mu$. The pulse self-distribution function $g(q, v, \sigma, t \,|\, \bar{q}, \bar{v}, \bar{\sigma}, \bar{t} = 0)$, which was defined in the previous section, is the kernel of the transformation which corresponds to the time t.

3.2. Reduced Description

By the spatial and temporal invariance of the infinite-volume equilibrium states of our system we have

$$g(q, v, \sigma, t \,|\, \bar{q}, \bar{v}, \bar{\sigma}, \bar{t}) = g(q - \bar{q}, v, \sigma, t - \bar{t} \,|\, 0, \bar{v}, \bar{\sigma}, 0) \tag{1}$$

To compute $g(q, v, \sigma, t \,|\, 0, \bar{v}, \bar{\sigma}, 0)$, it is convenient to use the *reduced description* for the configurations in Ω. To define it, we label the particles at the time $\bar{t} = 0$ so that $q_i \leqslant q_j$ for $i < j$ and $q_{-1} < 0 \leqslant q_0$. The reduced distance between particles is the length of space between them which is not covered by any rod. The reduced position of the nth particle x_n is defined by:

(1) $x_0 = q_0$.
(2) $x_n - x_0$ is the reduced distance between the nth and zeroth particles.

It is now easy to characterize, for the uncorrelated states, the conditional distribution on Ω given that there is a particle at the origin with $(\bar{q} = 0, \bar{v}, \bar{\sigma})$. The corresponding configurations, viewed as countable collections of points in Γ, are distributed independently in disjoint regions of Γ. They all include the point $(0, \bar{v}, \bar{\sigma})$ and the number of other points in any region has the Poisson distribution whose mean has the density $h(v)n_\sigma$, with

$$n_\sigma = \rho_\sigma \Big/ \Big(1 - \sum_\sigma \rho_\sigma d_\sigma\Big) \tag{2}$$

We define the functions D_n on Ω by

$$q_n = x_n + D_n, \qquad \forall n \in \mathbb{Z} \tag{3}$$

Using $\{D_n\}$, we now define the reduced positions at any time t, $\{x_n(t)\}$, by the above equation with q_n replaced by $q_n(t)$.

The resulting time evolution looks, in the reduced description, like the evolution of a system of equal-mass, impenetrable point particles, which carry an additional degree of freedom $-\sigma$. (See Fig. 1.) The reader is cautioned, however, that the labeling of particles cannot be done in a time-

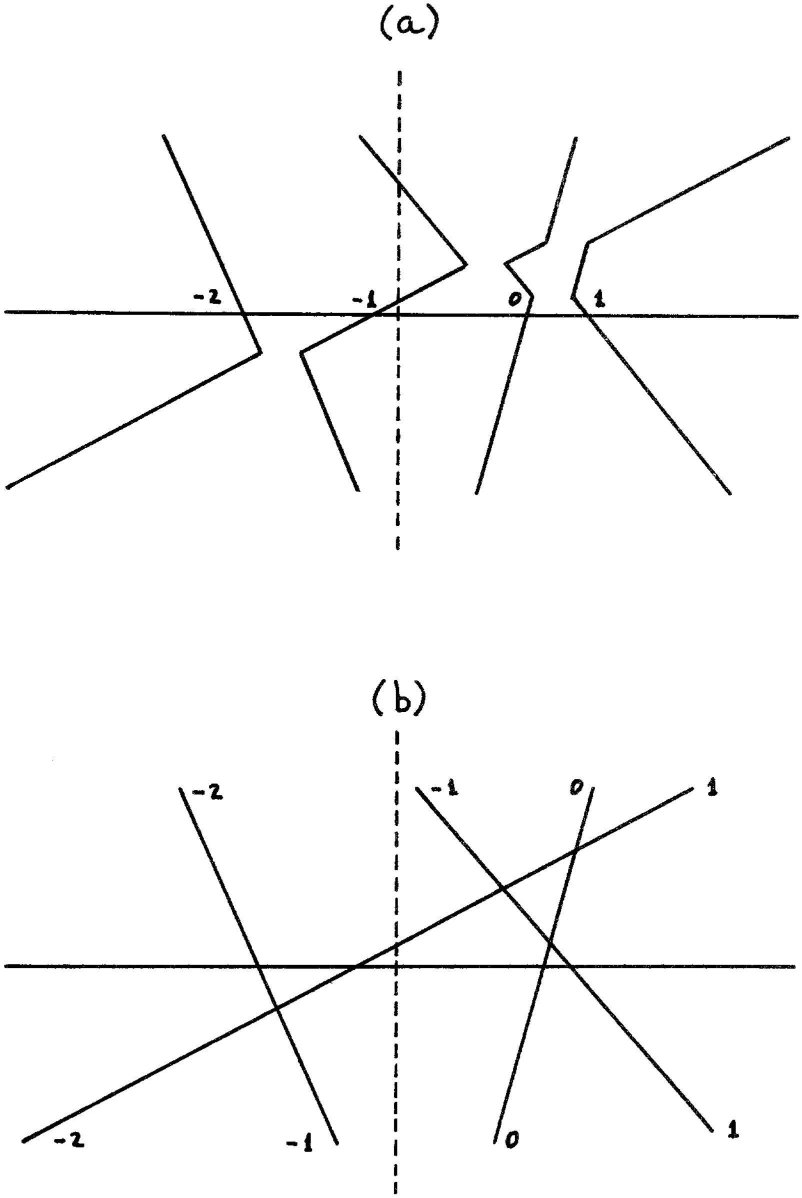

Fig. 1. (a) Space–time trajectories of the particles. (b) Space–time trajectories in the reduced description.

invariant method, which poses some difficulties in using this representation. (See Ref. 6.)

3.3. Pulse Self-Distribution Function

It is easy to see that the Fourier transform, in q, of $g(q, v, \sigma, t\,|\bar{q} = 0, \bar{v}, \bar{\sigma}, \bar{t} = 0)$ is

$$\delta(v - \bar{v})\, E(\chi_\sigma \exp[ikq(t)]|\bar{v}, \sigma) \equiv \delta(v - \bar{v})\, \hat{g}(k, \sigma, t\,|\bar{v}, \bar{\sigma}) \tag{4}$$

Here $\chi_\sigma(t)$ is the characteristic function that corresponds to the specified value of the parameter σ of the test pulse at the time t and $q(t)$ is its position. $E(\cdot\,|\bar{v}, \bar{\sigma})$ denotes the expectation value in the uncorrelated state in which the test pulse is at $(\bar{q} = 0, \bar{v}, \bar{\sigma})$.

Clearly, $q(t) = \bar{v}t + D_{M(t)}$, where $M(t)$ is the index of the particle on which the test pulse is located at the time t. We may compute the above expectation by first conditioning on M, thus:

$$\begin{aligned} &E(\chi_\sigma \exp[ikq(t)]|\bar{v}, \bar{\sigma}) \\ &\quad = [\exp(ik\bar{v}t)]E(E(\chi_\sigma(t) \exp[ikD_{M(t)}]|M(t), \bar{v}, \bar{\sigma})|\bar{v}, \bar{\sigma}) \end{aligned} \tag{5}$$

Now

$$D_M = \begin{cases} a_0/2 + a_1 + \cdots + a_{M-1} + a_M/2, & M > 0 \\ 0, & M = 0 \\ -(a_0/2 + a_{-1} + \cdots + a_{M+1} + a_M/2), & M < 0 \end{cases} \tag{6}$$

with $a_n = d_{\sigma_n}$, the diameter of the nth particle.

It follows from the above characterization of the uncorrelated states that

$$\begin{aligned} &E(\chi_\sigma(t) \exp[ikD_{M(t)}]|M(t), \bar{v}, \bar{\sigma}) \\ &\quad = \begin{cases} (\rho_\sigma/\rho) \exp[ik(d_\sigma + d_{\bar{\sigma}})/2] \exp\{(M - 1)[a(k) + ib(k)]\}, & M > 0 \\ \delta_{\sigma,\bar{\sigma}}, & M = 0 \\ (\rho_\sigma/\rho) \exp[-ik(d_\sigma + d_{\bar{\sigma}})/2] \exp\{(|M| - 1)[a(k) - ib(k)]\}, & M < 0 \end{cases} \\ &\quad \equiv f(M, k, \sigma, \bar{\sigma}) \end{aligned} \tag{7}$$

with $a(\cdot)$ and $b(\cdot)$ defined by

$$e^{a(k)+ib(k)} = \langle e^{ikd} \rangle = \sum (\rho_\sigma/\rho) e^{ikd_\sigma} \tag{8}$$

$M(t)$ equals the total number of collisions undergone by the test pulse in the time interval $[0, t]$, counting as positive (negative) those with pulses to its right (left). Thus $M = M_+ - M_-$, with $M_\pm$ being the numbers of pulses, in the reduced description, in the regions

$$\{(q, v, \sigma) \in \Gamma | q \underset{(<)}{>} 0, \quad q + t(v - \bar{v}) \underset{(>)}{<} 0\} \tag{9}$$

These are independently distributed with Poisson distributions.

Using Fourier transforms, we find that

$$E(f(M, k, \sigma, \bar{\sigma})|\bar{v}, \bar{\sigma}) = \frac{1}{2\pi}\int_0^{2\pi} d\tau\, \hat{f}(\tau, k, \sigma, \bar{\sigma})E(\exp[i\tau(M_+ - M_-)]|\bar{v}) \quad (10)$$

with

$$\begin{aligned}\hat{f}(\tau, k, \sigma, \bar{\sigma}) &= \sum_{l=-\infty}^{\infty} [\exp(-il\tau)]f(l, k, \sigma, \bar{\sigma}) \\ &= \delta(\sigma, \bar{\sigma}) \\ &\quad + \frac{\rho_\sigma}{\rho}\left\{\frac{\exp\{i[k(d_\sigma + d_{\bar{\sigma}})/2 - \tau]\}}{1 - \exp\{a(k) + i[b(k) - \tau]\}} + \text{comp. conj.}\right\} \quad (11)\end{aligned}$$

The contributions from M_+ and M_- factorize and we obtain, using (4),

$$\hat{g}(k, \sigma, t|\bar{v}, \bar{\sigma}) = [\exp(ikt\bar{v})]\frac{1}{2\pi}\int_0^{2\pi} d\tau\, \hat{f}(\tau, k, \sigma, \bar{\sigma})F(nt, \tau, \bar{v}) \quad (12)$$

with

$$\begin{aligned}F(nt, \tau, v) &= E(\exp[i\tau(M_+ - M_-)]|v) = \exp\{-n|t|[\mu(v)(1 - \cos\tau) \\ &\qquad - i\,\mathrm{sgn}(t)(v - v_0)\sin\tau]\} \\ \mu(v &= \int du\, h(u)|u - v| \quad (13) \\ v_0 &= \int du\, h(u)u, \qquad v_0 = 0 \quad \text{for } h(u) \text{ even}\end{aligned}$$

When only particles of one type are present we have

$$a(k) = 0, \qquad b(k) = kd, \qquad \text{and } \hat{f} = \delta(\tau - kd) \quad (14)$$

4. THE TOTAL DISTRIBUTION FUNCTION

Let I: $\Gamma \times \Omega \to \Omega$ be the natural imbedding under which the test pulse becomes indistinguishable from the other pulses. The total distribution function $f(q, v, \sigma, t|\bar{q}, \bar{v}, \bar{\sigma}, \bar{t} = 0)$ is the probability density of finding, at the time t, a particle at (q, v, σ) in the image, under I, of the uncorrelated state with the test pulse at $(\bar{q}, \bar{v}, \bar{\sigma})$.

Since our system has decaying correlations both in position and time[7] it follows that for any $\bar{q}$ f converges, as $q \to \infty$, or as $t \to \infty$, to $\rho_\sigma h(v)$. We denote by $\eta(k, v, \sigma, t|\bar{v}, \bar{\sigma})$ the Fourier transform in q (in the distributional sense) of

$$\eta(q, v, \sigma, t|\bar{v}, \bar{\sigma}) \equiv f(q, v, \sigma, t|\bar{q} = 0, \bar{v}, \bar{\sigma}, \bar{t} = 0) - \rho_\sigma h(v)$$

It is easy to see that, in terms of the reduced description,

$$\eta(k, v, \sigma, t \mid \bar{v}, \bar{\sigma}) = \int_{-\infty}^{\infty} P(dx, dv \mid \bar{v}, \bar{\sigma}) E(\chi_\sigma e^{ikq(t)} \mid x, v; 0, \bar{v}, \bar{\sigma}) \tag{15}$$

Here $P(dx, dv|\bar{v}, \bar{\sigma})$ is the probability of finding a pulse in the "region" $dx\,dv$ given that there is a pulse at $(\bar{q} = 0, \bar{v}, \bar{\sigma})$, and $E(\cdot|\cdot)$ denotes the expectation value for a pulse that is known to be at (x, v) at $t = 0$, with the additional knowledge that there is a pulse at $(\bar{q} = 0, \bar{v}, \bar{\sigma})(t = 0)$.

For the uncorrelated state

$$P(dx, dv|\bar{v}, \bar{\sigma}) = [\delta(x)\,\delta(v - \bar{v}) + nh(v)]\,dx\,dv \tag{16}$$

Let $M(t)$ denote the index of the particle on which the above-mentioned pulse is located at the time t. Then, in Eq. (15)

$$q(t) = x + tv + D_{M(t)} \tag{17}$$

To find the distribution of $M(t)$, notice that in any configuration it equals the number of collisions that an imaginary pulse of velocity $u = v + x/t$ would undergo starting at $x = 0$. Its collision with the pulse v should be counted only if it is positive and with $\bar{v}$ only if it is negative (this is easily seen using Fig. 2). Therefore the variable $\bar{M}$ defined by

$$M(t) = \bar{M} - \Theta(\bar{v} - u) + \Theta(u - v) \tag{18}$$

$$\Theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leqslant 0 \end{cases}$$

has the same distribution as the variable M discussed in Section 3, which corresponds to the velocity u.

Repeating the procedure used in the last section, we obtain

$$\eta(k, v, \sigma, t \mid \bar{v}, \bar{\sigma}) = (1/2\pi) \int_0^{2\pi} d\tau\, \hat{f}(\tau, k, \sigma, \bar{\sigma}) \int_{-\infty}^{\infty} dx\, [nh(v) + \delta(x - vt)\,\delta(v - \bar{v})] \times [\exp(ikx)] F(nt, \tau, x/t) \exp\{i\tau[\Theta(x - vt) - \Theta(\bar{v}t - x)]\} \tag{19}$$

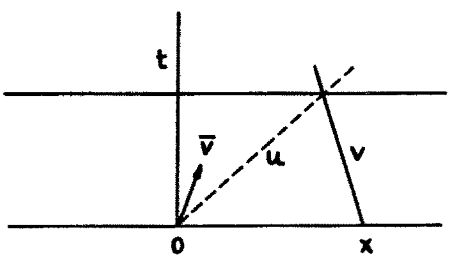

Fig. 2. The reduced trajectory of the imaginary pulse. $\bar{M} - \Theta(\bar{v} - u)$ is the index of the particle to its left.

This is in agreement with the result of Ref. 3, as seen by substituting $\hat{f} = \delta(\tau - kd)$, which corresponds to the system of identical rods, i.e., when $\rho_\sigma = 0$ except for one value of σ.

We find that the expression for η in a mixture has a strikingly simple relation to its value in a one-component system. Let us denote by $\eta_\tau^0(k, v, t\,|\bar{v})$ the value of the second integral in (19). For any given k it gives the function η for a one-component system of rods with diameter $d = \tau/k$. We can rewrite (19) as

$$\eta(k, v, \sigma, t\,|\bar{v}, \bar{\sigma}) = (1/2\pi)\int_0^{2\pi} d\tau\, \hat{f}(\tau, k, \sigma, \bar{\sigma})\eta_\tau^0(k, v, t\,|\bar{v}) \tag{20}$$

i.e., η is given as a time-independent average of η^0 over various diameters. The "weight function" $\hat{f}$ satisfies the normalization condition

$$(1/2\pi)\int_0^{2\pi} d\tau \sum_\sigma \hat{f}(\tau, k, \sigma, \bar{\sigma}) = 1 \tag{21}$$

$\hat{f}$ as a function of τ reflects the additional smoothing that results from the variability of the diameters. It is smooth (analytic) except for systems in which

$$d_\sigma \equiv d$$

e.g., if the various particles differ only by color or flavor. For such systems

$$\hat{f}(\tau, k, \sigma, \bar{\sigma}) = [\delta(\sigma, \bar{\sigma}) - \rho_\sigma/\rho] + (\rho_\sigma/\rho)\,\delta(\tau - kd) \tag{22}$$

When (22) is substituted into (20) we obtain, for $d_\sigma = d$,

$$\begin{aligned}\eta(k, v, \sigma, t\,|\bar{v}, \bar{\sigma}) = {} & \tilde{f}_s(k, v, t\,|\bar{v})[\delta(\sigma, \bar{\sigma}) - \rho_\sigma/\rho] \\ & + (\rho_\sigma/\rho)\eta^0(k, v, t\,|\bar{v})\end{aligned} \tag{23}$$

Here $\tilde{f}(k, v, t\,|\bar{v})$ is the Fourier transform of $f_s(q, v, t\,|0, \bar{v}, 0)$ and $\eta^0(k, v, t\,|\bar{v})$ is the value of η in a one-component system with diameter d and density ρ.

5. DENSITY CORRELATION FUNCTIONS

Our results may be used to compute the van Hove time-dependent two-particle density correlation function. Let $G(r, t, \sigma, \bar{\sigma})$ be the probability density, in an equilibrium state, of finding in two observations, separated by a time t, particles each in a specified position with separation r. Clearly,

$$G(r, t, \sigma, \bar{\sigma}) = \int dv \int d\bar{v}\, f(r, v, \sigma, t\,|0, \bar{v}, \bar{\sigma}, \bar{t} = 0)\rho_{\bar{\sigma}} h(\bar{v}) \tag{24}$$

Its space–time Fourier transform, which for real systems can be measured by coherent neutron scattering, is

$$S(k, \omega, \sigma, \bar{\sigma}) = (1/2\pi) \int dr \int dt\, e^{i(kr - \omega t)}[G(r, t, \sigma, \bar{\sigma}) - \rho_\sigma \rho_{\bar{\sigma}}]$$
$$= (1/2\pi) \int dt\, e^{-i\omega t} \int dv \int d\bar{v}\, \eta(k, v, \sigma, t \,|\, \bar{v}, \bar{\sigma}) \rho_{\bar{\sigma}} h(\bar{v}) \quad (25)$$

Our explicit result for η is in a simple relation, given by Eq. (20), to the corresponding function for a system of identical rods. This relation holds for all quantities that, like G and S, depend linearly on η. For example, the Fourier–Laplace transform of G,

$$\hat{\chi}(k, s, \sigma, \bar{\sigma}) = \int_0^\infty dt\, e^{-st} \int_{-\infty}^\infty dr\, e^{ikr}[G(r, t, \sigma, \bar{\sigma}) - \rho_\sigma \rho_{\bar{\sigma}}] \quad (26)$$

can be brought to the form

$$\begin{aligned}
\hat{\chi}&(k, s, \sigma, \bar{\sigma}) \\
&= \frac{1}{2\pi} \int_0^{2\pi} d\tau\, \hat{f}(\tau, k, \sigma, \bar{\sigma})(-k^2) \\
&\quad \times \int_{-\infty}^\infty dv\, \frac{h(v)}{[\alpha(\tau)\mu'(v) - i\beta(k, \tau)]^2 [S - i\beta(k, \tau)(v - v_0) + \alpha(\tau)\mu(v)]} \\
&= \frac{1}{2\pi} \int_0^{2\pi} d\tau\, \hat{f}(\tau, k, \sigma, \bar{\sigma}) \frac{k^2}{2\alpha(\tau)} \\
&\quad \times \int_{-\infty}^\infty \frac{dv}{[s - i\beta(k, \tau)(v - v_0) + \alpha(\tau)\mu(v)]^2}
\end{aligned} \quad (27)$$

with

$$\alpha(\tau) = n(1 - \cos \tau), \qquad \beta(k, \tau) = k + n \sin \tau \quad (28)$$

$\mu'(v)$ is discontinuous at the points at which $h(v)$ has δ-function singularities. At such points the discontinuous part in the denominator in (27) should be interpreted as

$$[\alpha(\tau)\mu'(v+) - i\beta(k, \tau)][\alpha(\tau)\mu'(v-) - i\beta(k, \tau)]$$

The derivation of Eq. (27) follows closely that of Eqs. (2.23) and (2.19) of Ref. 3 [which are recovered by the substitution $\hat{f}(\tau, k, \sigma, \bar{\sigma}) = \delta(\tau - kd)$]. Use is made there of the relation

$$\mu''(v) = 2h(v) \quad (29)$$

S may be obtained by

$$S(k, \omega, \sigma, \bar{\sigma}) = (1/\pi) \operatorname{Re} \hat{\chi}(k, i\omega, \sigma, \bar{\sigma}) \quad (30)$$

In systems of rods of constant diameter the presence of discrete velocities [i.e., δ-singularities in $h(v)$] results, as was noted in Ref. 3, in damped oscillation modes in $\chi(k, t)$. For any system in which the diameters are not strictly constant this feature is erased by the averaging with respect to τ.

6. KINETIC BEHAVIOR

In Ref. 3, LPS studied the time evolution of the function f for a one-component system of hard rods. They denote by $T(t)$ the operator whose kernel, in the (x, v) space, is given by $f(\cdot, t|\cdot)$. Now, $T(t)$ applied to the distribution of the test particle gives the one-particle correlation function, at the time t, in the corresponding uncorrelated state. $T(t)$ commutes with translations and in the Fourier-transform representation is given by operators $T_0(t; k, d)$ which act in the v space. By the subscript we indicate that the operator corresponds to a system of rods of a constant diameter d. It is easily seen from Eq. (20) that the corresponding operator in (v, σ) space is

$$T(t; k) = (1/2\pi) \int_0^{2\pi} d\tau\, R(k, \tau) T_0(t; k, \tau/k) \tag{31}$$

where $R(k, \tau)$ is the operator, in the σ space, whose kernel is $\hat{f}(\tau, k, \sigma, \bar{\sigma})$ and T_0 acts, as explained above, only on the v component.

LPS[3] found that, surprisingly,

$$T_0(t; k, d) = [\exp(itB_{k,d})] T_0(0; k, d) \tag{32}$$

with a time-independent collision operator B.

This property no longer holds for general systems considered by us, as may be guessed from the above expression for $T(t; k)$.

A similar difference occurs in the kinetic behavior of the pulse self-distribution. This, however, is a simple consequence of the fact that the motion of a single pulse is Markovian in the system of equal diameters, where subsequent collisions of a pulse are independent, and non-Markovian if the acquired information about the diameters of other particles becomes nontrivial.

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